# Homework Assignment (Problem Set) 2:

Note, Problem Set 2 directly focuses on Modules 3 and 4: Linear Programming and the Economic Interpretation of the Dual and Sensitivity Analysis, and Network Models.

***4 Questions***

Rubric:

All questions worth 37.5 points

37.5 Points: Answer and solution are fully correct and detailed professionally.

25-37 Points: Answer and solution are deficient in some manner but mostly correct.

15-24 Points: Answer and solution are missing a key element or two.

1-14 Points: Answer and solution are missing multiple elements are significantly deficient/incomprehensible.

0 Points: No answer provided.

Question 1:

**1A.** Write the general dual problem associated with the given LP.

*(Do not transform or rewrite the primal problem before writing the general dual)*

Maximize –4x1 + 2x2

Subject To

4x1 + x2 + x3 = 20

2x1 – x2 ≥ 6

x1 – x2 + 5x3 ≥ –5

–3x1 + 2x2 + x3 ≤ 4

x1 ≤ 0, x2 ≥ 0, x3 unrestricted

**1B.** Given the following information for a product-mix problem with three products and three resources.

**Primal Decision Variables:** x1 = number of unit 1 produced; x2 = # of unit 2 produced; x3 = # of unit 3 produced

**Primal Formulation: Dual Formulation:**

Max Z (Rev.) = 25x1 + 30x2 + 20x3 Min W = 50π1 + 20π2 +25π3

Subject To 8x1 + 6x2 + x3 ≤ 50 (Res. 1 constraint) Subject To 8π1 + 4π2 +2π3≥ 25

4x1 + 2x2 + 3x3 ≤ 20 (Res. 2 constraint) 6π1 + 2π2 +π3 ≥ 30  
 2x1 + x2 + 2x3 ≤ 25 (Res. 3 constraint) π1 + 3π2 +2π3≥ 20

x1, x2, x3 ≥ 0 (Nonnegativity) π1, π2, π3 ≥ 0

**Optimal Solution:**

Optimal Z = Revenue = $268.75

x1 = 0 (Number of unit 1) Dual Var. Optimal Value = 22.5 (Surplus variable in 1st dual constraint)

x2 = 8.125 (Number of unit 2) Dual Var. Optimal Value = 0 (Surplus variable in 2nd dual constraint)

x3 = 1.25 (Number of unit 3) Dual Var. Optimal Value = 0 (Surplus variable in 3rd dual constraint)

Resource Constraints:

Resource 1 = 0 leftover units Dual Var. Optimal Value = 3.125 = π1

Resource 2 = 0 leftover units Dual Var. Optimal Value = 5.625 = π2

Resource 3 = 14.375 leftover units Dual Var. Optimal Value = 0 = π3

***1Bi.*** What is the fair-market price for one unit of Resource 3?

***1Bii.*** What is the meaning of the surplus variable value of 22.5 in the 1st dual constraint with respect to the primal problem?

Question 2:

Carco manufactures cars and trucks. Each car contributes $300 to profit and each truck, $400; these profits do not consider machine rental. The resources required to manufacture a car and a truck are shown below. Each day Carco can rent up to 98 Type 1 machines at a cost of $50 per machine. The company now has 73 Type 2 machines and 260 tons of steel available. Marketing considerations dictate that at least 88 cars and at least 26 trucks be produced.

Part A: Formulate the problem as a Linear Program.

Part B: Solve the LP (provide exact values for all variables and the optimal objective function).

*Hint: The optimal objective function value is $32540*

*[Note, I am providing this hint because having the optimal solution is necessary to do Part C.]*

Part C: Answer the following questions from your output. *(Note, do not simply rerun the model – use the Linear Programming output and Sensitivity Analysis to explain your answers.)*

i) If cars contributed $310 to profit, what would be the new optimal solution to the problem?

ii) What is the most that Carco should be willing to pay to rent an additional Type 1 machine for 1 day?

iii) What is the most that Carco should be willing to pay for an extra ton of steel?

iv) If Carco were required to produce at least 86 cars, what would Carco’s profit become?

v) Carco is considering producing jeeps. A jeep contributes $600 to profit and requires 1.2 days on machine 1, 2 days on machine 2, and 4 tons of steel. Should Carco produce any jeeps?

Table:

Vehicle Type Days on Machine 1 Days on Machine 2 Tons of Steel

Car 0.8 0.6 2

Truck 1 0.7 3

Question 3:

A catering company must have the following number of clean napkins available at the beginning of each of the next four days: day 1: 15, day 2: 12, day 3: 18, and day 4: 6. After being used, a napkin can be cleaned by one of two methods: fast service or slow service. Fast service costs $0.10 per napkin, and a napkin cleaned via fast service is available for use the day after it is last used. Slow service costs $0.06 per napkin, and a napkin cleaned via slow service is available two days after they were last used. New napkins can be purchased for a cost of $0.20 per napkin.

Part A: Formulate the problem as a minimum cost transportation problem.

Part B: Solve the problem (provide exact values for all variables and the optimal objective function).

Question 4:

A university has three professors who each teach four courses per year. Each year, four sections of marketing, finance, and production must be offered. At least one section of each class must be offered during each semester (fall and spring). Each professor’s time preferences and preference for teaching various courses are given below.

The total satisfaction a professor earns teaching a class is the sum of the semester satisfaction and the course satisfaction. Thus, professor 1 derives a satisfaction of 3 + 6 = 9 from teaching marketing during the fall semester.

Part A: Formulate the problem as a minimum cost network flow problem that can be used to assign professors to courses so as to maximize the total satisfaction of the three professors. Draw the network and identify the nodes and arcs.

Part B: Solve the problem (provide exact values for all variables and the optimal objective function).

Table:

Professor 1 Professor 2 Professor 3

Fall Preference 3 5 4

Spring Preference 4 3 4

Marketing 6 4 5

Finance 5 6 4

Production 4 5 6